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OPTIMIZATION OF TIME-TEMPERATURE PARAMETERS FOR CREEP AND STRESS RUPTURE, WITH APPLICATION TO DATA FROM GERMAN COOPERATIVE LONG-TIME CREEP PROGRAM

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SUMMARY

By the use of orthogonal polynomials developed for discrete sets of data, the least-squares equations for determining the optimized stress-rupture parametric constants are obtained in nearly uncoupled form; thus the use of high-degree polynominals is permitted without the loss of significant figures. Optimum values of the constants can thereby be accurately obtained. The method is applied to the data obtained from the German cooperative long-time creep program by using a general parameter of which the Manson-Haferd and Larson-Miller parameters are special cases. Good correlation was obtained. An analysis is also made of creep data obtained for columbium alloy FS-85 with good results. A complete Fortran IV computer program is included to aid those wishing to use the method.

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INTRODUCTION

One method of extrapolating short-time creep-rupture data to predict long-time life involves the use of a time-temperature parameter. This concept is based on the assumption that all creep-rupture data for a given material can be correlated to produce a single "master curve" wherein the stress (or log stress) is plotted against a parameter involving a combination of time and temperature. Extrapolation to long times can then be obtained from this master curve, which can presumably be constructed by using only short-time data. Three well-known parametric methods are the Larson-Miller, Manson-Haferd, and Dorn parameters (refs. 1 to 3). These parametric methods have the great advantage, at least in theory, of requiring only a relatively small amount of data to establish the required master curve.

All

More recently a general creep-rupture parameter was introduced by one of the authors (ref. 4) that includes most of the currently used parameters as special cases. The analysis in the present paper is therefore based on this general parameter.

A significant advance in the practical application of the parametric methods was the development of an objective least-squares method for determining the optimum values of the parametric constants without plotting and cross-plotting the data and without the use of judgment on the part of the analyst (ref. 5). This least-squares method involves, however, several practical difficulties that arise from the fact that in fitting the master curve by a polynomial, the set of linear algebraic equations for the coefficients (the normal equations) are very ill-conditioned. The determinant of these equations can be shown to be related to the Hilbert determinant (ref. 6), which rapidly approaches zero as its order increases. Thus for polynomials above the second degree, it is necessary to use double-precision arithmetic (16 significant digits or more) on the computer, and for the fifth degree and above the results become uncertain even with double-precision arithmetic. This difficulty is inherent in the normal least-squares equations and is not limited only to the stress-rupture problem.

The present report presents a method for avoiding the above difficulty by using orthogonal polynomials in the representation of the master curve (appendix A). The use of orthogonal polynomials for representing discrete sets of unequally spaced data is described in reference 6 and in more detail in reference 7. A further improvement can be obtained by performing a linear transformation on the stresses (or the logs of the stresses) so that all the values of stress (or log stress) lie between 2 and -2, as recommended in reference 7. As a result of these innovations, it became possible to perform all the computations in single-precision arithmetic (eight significant digits) up to 18th degree polynomials without appreciable round-off error.

In addition, this report contains a complete analysis, in which the general parameter was used, of all the data for three steels that were obtained by NASA through the cooperation of Dr. K. Richard of Faberwerke Hoechst in Frankfurt and that were investigated in a long-time cooperative creep program in Germany. Some of the data from the latter investigation are included in this paper.

Finally it is shown by means of a concrete example how the parameter techniques can be applied to creep data to predict long-time creep. For this purpose the data for columbium alloy FS-85, as reported in reference 8, are used.

A complete Fortran IV program, as used on the IBM 7094 computer in making the calculations, is presented in appendix B. This program can be used for the objective analysis of any set of creep-rupture data by the Larson-Miller, Manson-Haferd, or the more general parameter of reference 4.

SYMBOLS

A,B linear transformation coefficients

a,b,c elements of coefficient matrix

D standard deviation

K	degree of freedom
m	degree of polynomial
n	number of data points
$P(\sigma)$	creep-rupture parameter
Q	polynomial
q	stress exponent
r	temperature exponent
S	sum of squares of residuals
T	temperature
T_a	temperature intercept
t	time to rupture
t_a	time intercept
u	coefficient of polynomial function
X	scaled log stress
x	log stress
y	log time
y_a	log time intercept
α,β	constants from recurrence relation
σ	stress
τ	$\sigma^q(T - T_a)^r$

Subscripts:

max	maximum
min	minimum

PROCEDURE

General Parameter

The general creep-rupture parameter introduced in reference 4 has the fol-

lowing form

$$P(\sigma) = \frac{\dfrac{\log t}{\sigma^q} - \log t_a}{(T - T_a)^r} \tag{1}$$

where T_a, $\log t_a$, q, and r are material constants to be determined from the available experimental data. The parameter $P(\sigma)$ is a function of the stress and, when plotted against stress, is referred to as a master curve (fig. 1, p. 9). If $q = 0$ and $r = 1$, the Manson-Haferd parameter is obtained. If $q = 0$, $r = -1$, and $T_a = -460^\circ$ F, the Larson-Miller parameter results. If $q = 1$ and $r = 1$, the stress-modified parameter suggested in reference 9 is obtained. Finally, if $q = 0$, equation (1) reduces to the parameter proposed by Manson and Brown (ref. 10).

The object is to find the best values of the constants q, $\log t_a$, T_a, and r so that the master curve best fits the data. To find these values, the method of least squares is used whereby the master curve is represented by a polynomial in the logarithm of the stress, and the best fit is obtained by minimizing the sum of the squares of the deviations (the residuals) of the data from the curve. The calculation procedure will now be described. The details of the derivation are given in appendix A, and a Fortran IV computer program using this method is given in appendix B.

Calculation Procedure

To simplify the notation, the following symbols are introduced:

$$\left.\begin{aligned} \tau &\equiv \sigma^q (T - T_a)^r \\ y &\equiv \log t \\ x &\equiv \log \sigma \\ y_a &\equiv \log t_a \end{aligned}\right\} \tag{2}$$

Then from equation (1) it follows that

$$y = \sigma^q y_a + \tau Q(x) \tag{3}$$

where in reference 5, $Q(x)$ was represented by a simple polynomial of the form

$$Q(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_m x^m \tag{4}$$

The least-squares equations obtained sometimes led to difficulties as indicated in the INTRODUCTION. These difficulties can be avoided, however, by rewriting equation (4) in terms of polynomials that are orthogonal over the set of data, as defined in appendix A. Thus assume

$$Q(x) = u_1 Q_1(x) + u_2 Q_2(x) + \cdot \cdot \cdot + u_{m+1} Q_{m+1}(x) = \sum_{j=1}^{m+1} u_j Q_j(x) \tag{5}$$

where u_j is an unknown constant, m is the degree of the highest degree polynomial, and $Q_j(x)$ is a polynomial of degree $j - 1$ that satisfies the orthogonality conditions described in appendix A. The use of orthogonal polynomials permits the solution of the least-squares equations directly in closed form, thus the loss of a large number of significant digits is avoided. The method of calculating Q_j will be discussed in appendix A.

If equation (5) is substituted into equation (3), an equation with $m + 5$ unknown constants results for the case of the general parameter. For the case of the linear parameter there are $m + 3$ constants, and for the Larson-Miller parameter there are $m + 2$. It is necessary that the number of data points n always equals or exceeds the number of unknown constants.

The constants are determined so that equation (3) fits the data best in the least-squares sense. To accomplish this, the sum of the squares of the deviations is minimized; that is,

$$S \equiv \sum_{i=1}^{n} \left[y_i - \sigma_i^q y_a - \tau_i Q(x_i) \right]^2 \tag{6}$$

is made a minimum. Because the equations are nonlinear in some of the unknown constants a trial and error procedure must be used. A set of values is assumed for q, r, and T_a, and the corresponding best values of y_a and u_j are determined. A different set of values for q, r, and T_a is then chosen, and again the best values of y_a and u_j are calculated. Several sets of values of q, r, and T_a are tried, and the values corresponding to the overall best fit are determined. For the case of the linear parameter, only the value of T_a is varied (q is always equal to zero, and r is always equal to 1). For the Larson-Miller parameter, T_a is equal to -460° F, and no trial and error procedure is needed.

As a measure of the fit, the standard deviation D, defined by

$$D = \sqrt{\frac{S}{n - K}} \tag{7}$$

is used, where K equals

$$K = \left. \begin{array}{ll} m + 5 & \text{general parameter} \\ m + 3 & \text{linear parameter} \\ m + 2 & \text{Larson-Miller parameter} \end{array} \right\} \tag{8}$$

The smallest value of D will correspond to the best fit.

To determine the best values of y_a and u_j for a given set of values of T_a, q, and r, the following calculations are made. First, the logarithms of the stresses are scaled so that they lie in the range -2 to 2, as suggested in reference 7. The reason for this is discussed in appendix A. Thus define a variable X by

$$X = Ax + B \tag{9a}$$

$$\left.\begin{aligned} A &= \frac{4}{x_{max} - x_{min}} \\ B &= -2\,\frac{x_{max} + x_{min}}{x_{max} - x_{min}} \end{aligned}\right\} \tag{9b}$$

The polynomials $Q_j(X_i)$ are now calculated for each of the data points by using the following formulas:

$$Q_{j+1} = (X - \alpha_j)Q_j - \beta_j Q_{j-1} \qquad m \geq j \geq 1 \tag{10}$$

$$\left.\begin{aligned} \alpha_j &= \frac{\sum_{i=1}^{n} X_i \tau_i^2 Q_j^2(X_i)}{\sum_{i=1}^{n} \tau_i^2 Q_j^2(X_i)} \qquad m \geq j \geq 1 \\ \beta_j &= \frac{\sum_{i=1}^{n} X_i \tau_i^2 Q_j(X_i) Q_{j-1}(X_i)}{\sum_{i=1}^{n} \tau_i^2 Q_{j-1}^2(X_i)} \qquad m \geq j > 1,\ Q_1 = 1,\ \text{and}\ \beta_1 = 0 \end{aligned}\right\} \tag{10a}$$

where n is the number of data points, X_i is the scaled value of log for the i^{th} data point, and τ_i is equal to $\sigma_i^q(T_i - T_a)^r$ for the i^{th} data point for the chosen values of T_a, q, and r.

It is to be noted that the degree of the polynomial $Q(x)$ of equation (5) can be increased by merely computing the next polynomial in the series Q_{m+2} without having to recompute any of the previous ones. This is one of the advantages of using orthogonal polynomials.

Once the values of Q_j have been computed for each of the data points, y_a and u_j can be calculated as follows:

Let

$$\left.\begin{aligned}
a_0 &= \sum_{i=1}^{n} \sigma_i^{2q} \\
a_j &= \sum_{i=1}^{n} \sigma_i^{q}\tau_i Q_j(X_i) \\
b_j &= \sum_{i=1}^{n} \tau_i^2 Q_j^2(X_i) \\
c_0 &= \sum_{i=1}^{n} \sigma_i^{q} y_i \\
c_j &= \sum_{i=1}^{n} \tau_i y_i Q_j(X_i)
\end{aligned}\right\} \qquad (11)$$

where $j = 1,2 \ldots m + 1$.

Then

$$\left.\begin{aligned}
y_a &= \frac{c_0 - \sum_{j=1}^{m+1} \frac{a_j c_j}{b_j}}{a_0 - \sum_{j=1}^{m+1} \frac{a_j^2}{b_j}} \\
u_j &= \frac{c_j - a_j y_a}{b_j}
\end{aligned}\right\} \qquad (12)$$

Note that if $q = 0$, a_0 equals the number of data points n. Thus by means of equations (9) to (12), the best values of y_a and u_j to fit the data are found for a given choice of T_a, q, and r. The Fortran IV program described in appendix B automatically scans all the desired values of T_a, q, and r and chooses the best set from all the submitted values as determined by the smallest value of the standard deviation D, as defined by equation (7). The method can be illustrated by a simple example: consider a set of theoretical data, which fit the following equation exactly

$$\frac{9.5 - \log t}{T - 600} = 10^{-3}(7.02 + 0.467\ x + 0.061\ x^2 + 0.00928\ x^3) \tag{13}$$

Eight data points satisfying this equation are given in columns 2 to 6 of table I. For this data $T_a = 600^\circ$ F and $\log t_a = y_a = 9.5$. Suppose, however, that these eight data points were obtained experimentally and that the values of T_a and $\log t_a$ were not known. The problem then is to find the best values of T_a and $\log t_a$ to fit the data by the linear parameter. These values can readily be found by using the equations of the previous section. First, from column 6 of table I

$$(\log \sigma)_{max} = 4.75051$$

$$(\log \sigma)_{min} = 1.81954$$

Therefore from equations (9b)

$$A = 1.36474$$

$$B = -4.48319$$

and by means of equation (9a) the X_i were computed and are given in column 8.

For illustrative purposes three values of T_a were chosen, 500°, 600°, and 700° F. For each of these values of T_a, values of T_i, α_j, β_j, and $Q_j(X_i)$ were computed by means of equations (2), (10), and (10a), and the values of a_j, b_j, and c_j were computed by equations (11). The results are tabulated for $T_a = 600^\circ$ in columns 9 to 12 of table I and in table II up to a third degree polynomial.

The values of y_a and u_j were then computed by using equations (12) for each of these three values of T_a by first assuming $m = 2$, then $m = 3$, and finally $m = 4$, corresponding to polynomials of second, third, and fourth degrees, respectively. For each of these cases the standard deviation D was computed from equation (7) with S being given by equation (6) and Q by equation (5). The results are summarized in table III. The least value of D, signifying the best fit, is obtained for $m = 3$ and $T_a = 600^\circ$ F. The corresponding value of y_a is 9.5. These values, of course, correspond to equation (13), from which the data were generated.

Application to Data from German Cooperative Long-Time Creep Program

As part of the German cooperative long-time creep program, a sufficient amount of material of each of three steels was supplied to NASA to permit the running of short-time tests necessary to predict the results at long times obtained in the German test program. The composition of these steels is shown in table IV.

The results of the NASA tests, which were used in the subsequent analysis,

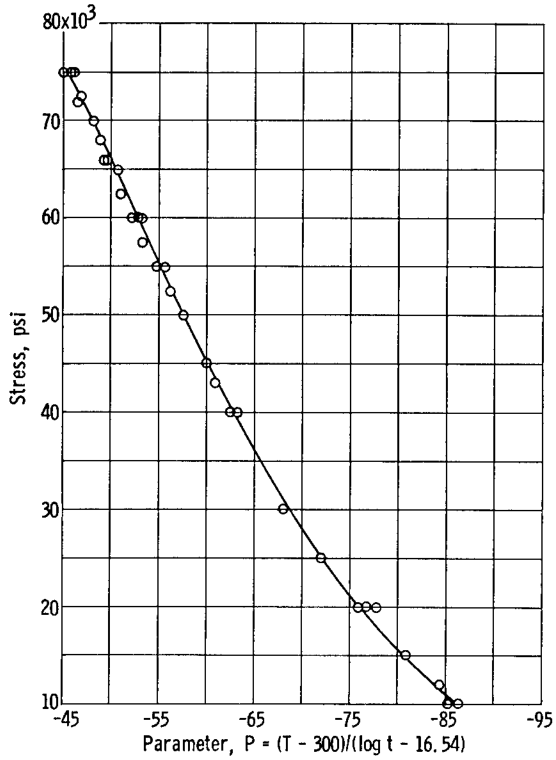


Figure 1. - Master curve for steel K (27b KK), calculated from NASA data between 10 and 3700 hours.

are shown in table V. Table VI shows the results of the long-time German test program. The three steels will be designated briefly as steel K, steel C, and steel P.

With the use of the test data shown in table V a complete analysis was made by the previously described method. The general parameter discussed in the INTRODUCTION was used, and the best values were obtained for the parametric constants for each of the three steels.

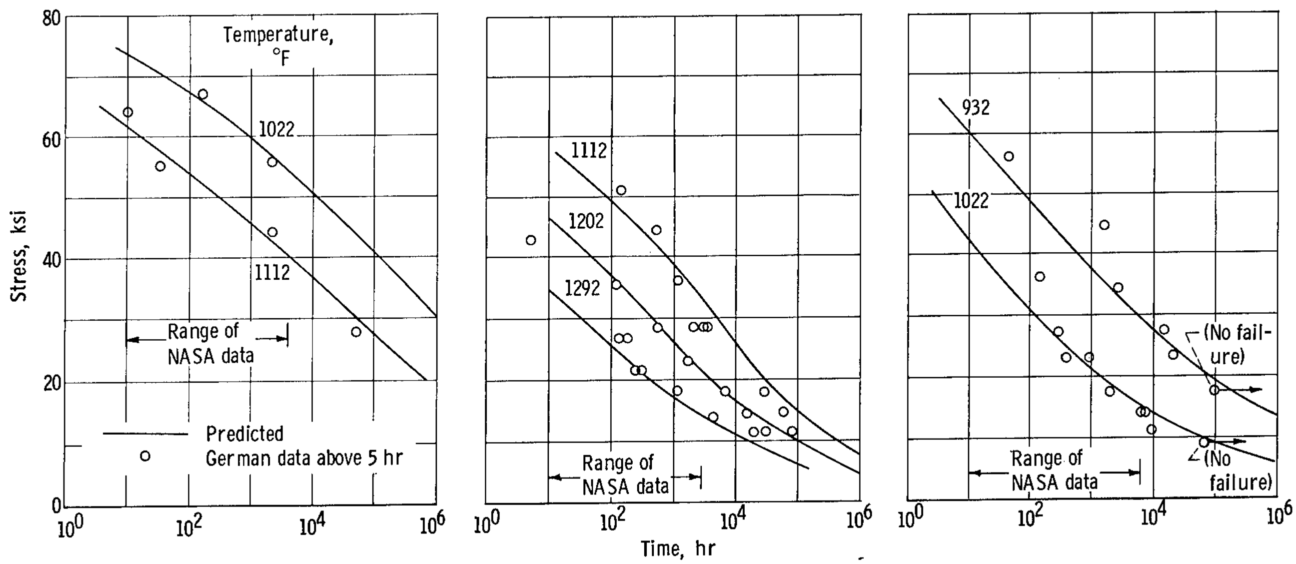
All the data obtained for these steels are shown in tables V and VI. Many of the data points were obtained for purposes other than the application to time-temperature parameters, as described in this report. As already discussed in references 4 and 11, a much smaller amount of data is needed when an accelerated program is desired; however, since these data were already available, all the data indicated in tables V and VI were used to obtain the best possible parametric constants.

For all three steels the analysis showed the stress exponent q to be zero, but the temperature exponent r to be different for each of the three materials. For steel K the best value of r was 1, which indicated that the best fit is obtained by the linear parameter. For steel P a value of r of -1 was obtained, which indicated a parameter similar to the Larson-Miller parameter; however, the corresponding value of T_a was 200° F rather than -460° F used in the Larson-Miller parameter. For steel C the value of R was 2.5.

Figure 1 shows the results for steel K. Here the master curve consists of a plot of stress against the optimized parameter $(T - 300)/(\log t - 16.54)$.

Figure 2 shows the isothermals computed by using the optimized parameters, as shown on each of the figures. The range of the NASA data used to obtain these parameters is also shown on each of the figures. The data points shown are the German results obtained to date. The predictions up to 100 000 hours from the NASA data based on the optimized parameters agree well with the German data, if scatter and differences in testing technique between the two organizations are considered.

Figure 3 shows a comparison for each of the three steels between the best linear parameter, the best Larson-Miller parameter, and the best general parameter. Although for some of the steels fair agreement can be obtained with one or the other of these parameters, it is clear that the general parameter is superior when all the materials are considered jointly. If any one of the special cases of this parameter is to be chosen for all materials, the linear

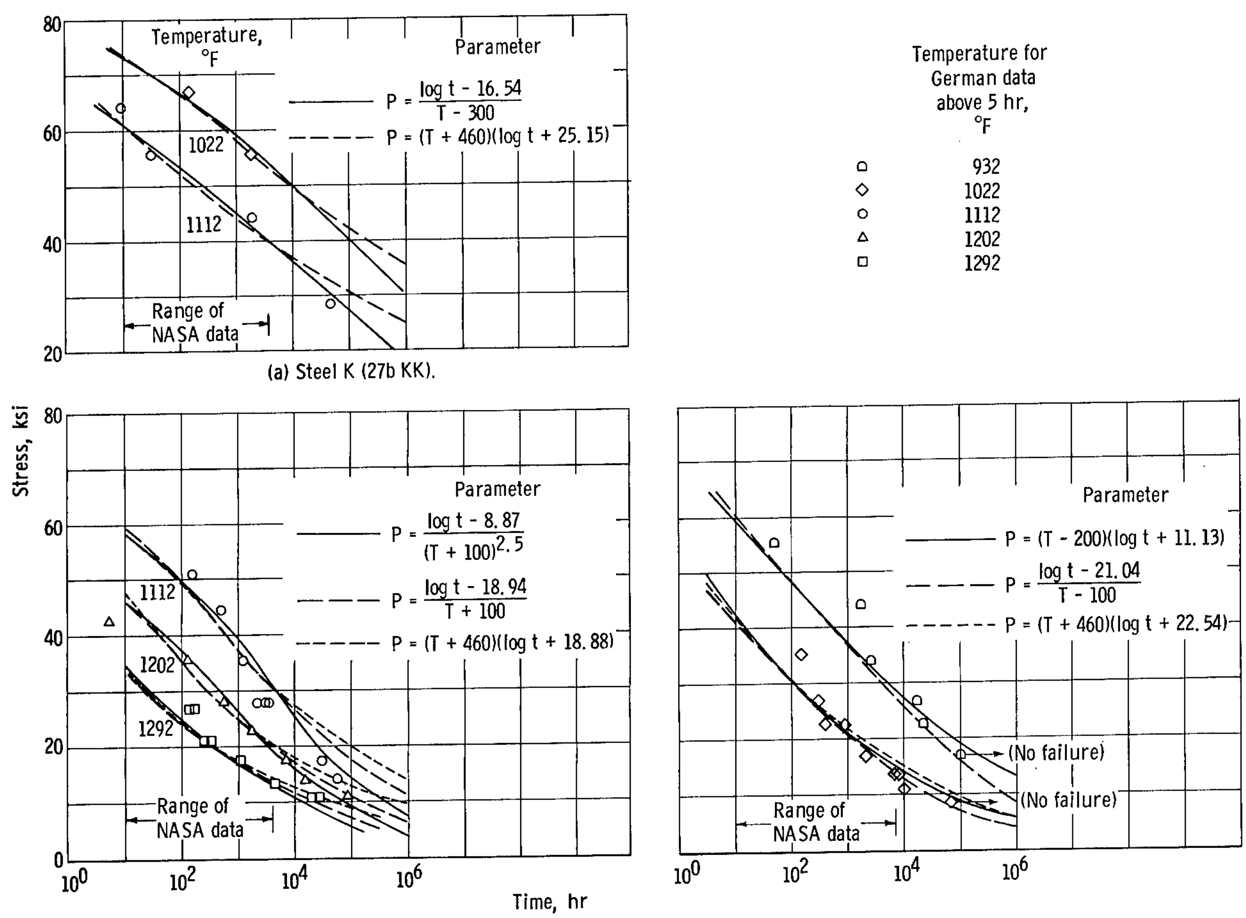


(a) Steel K (27b KK); parameter, P = (log t - 16.54)/(T - 300).

(b) Steel C (23b CK); parameter, P = (log t - 8.87)/(T + 100)$^{2.5}$.

(c) Steel P (14a PA); parameter, P = (T - 200)(log t + 11.13).

Figure 2. - Analysis of German steel data by generalized parameter with optimum constants (where T is temperature, and t is time to rupture).

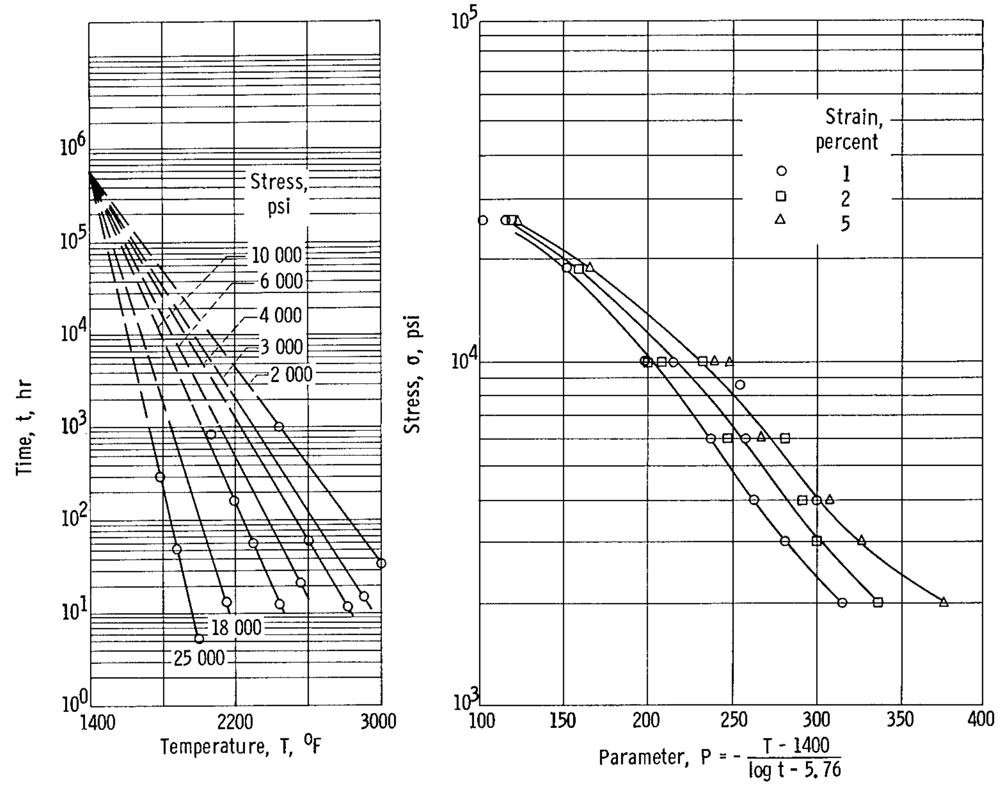


(a) Steel K (27b KK).

(b) Steel C (23b CK).

(c) Steel P (14a PA).

Figure 3. - Analysis of German steel data by several parameters (where T is temperature, and t is time to rupture).



(a) 5-Percent strain.

(b) Master curves obtained for 1-, 2-, and 5-percent strain.

Figure 4. - Analysis of creep data for columbium alloy FS-85 by linear parameter.

parameter would appear to be the best choice.

Application to Creep Data

Although there is no fundamental reason why the same parameter is capable of representing both creep and rupture data, it has nevertheless been found empirically (refs. 1 and 2) that the dual role of the same parameter leads to reasonable results. Experimental data for creep are much more limited, however, than that for rupture, and such data tend to contain more scatter; hence, analysis of creep data by the parametric approach has been limited in the past.

The method of the present report can be applied directly to creep data without any change. All that is necessary is to redefine t as the time to attain a specified amount of creep rather than as the rupture time. Thus, it is assumed that for a given amount of creep, say 1 percent, a plot of $\log \sigma$ against a parameter, such as that given by equation (1), will produce a single master curve. For a different amount of creep, say 5 percent, a different master curve can be obtained, but it is assumed that the parametric constants, such as $\log t_a$ and T_a, remain the same and that they equal the values obtained from rupture data.

Calculations of this type were performed for columbium alloy FS-85. The creep tests were limited to runs of approximately 1000 hours; the data are

given in table VII, as taken from reference 8. Figure 4(a) shows the data for 5-percent creep strain, and figure 4(b) shows the master curves obtained for 1-, 2-, and 5-percent strain as well as the parametric constants obtained by the method of this report. While scatter in the creep data is high, the correlation must be regarded as good. In general, the points agree well with the master curve.

Although these results are encouraging, much more work is necessary before it can be concluded that the parametric approach is completely valid for creep data. If it is eventually concluded that the parametric approach is valid for creep data and in particular that the parametric constants are the same for both the creep and rupture processes, it is obvious that a great saving in test facilities and test program planning will result. It therefore seems very worthwhile in future studies to give more attention to the correlation and extrapolation of creep data by the parametric method.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, May 3, 1965.

APPENDIX A

ORTHOGONAL POLYNOMIALS AND LEAST-SQUARES DETERMINATION OF PARAMETRIC CONSTANTS

A set of polynomials $Q_j(x)$ are said to be orthogonal over an interval with respect to the weighting function $\tau(x)$ if they satisfy the following relation

$$\int_{x=x_1}^{x=x_2} \tau^2(x)Q_j(x)Q_k(x)dx = 0 \qquad j \neq k \tag{A1}$$

Similarly a set of polynomials can be defined to be orthogonal over a set of n discrete points x_i by the following relation

$$\sum_{i=1}^{n} \tau_i^2 Q_j(x_i)Q_k(x_i) = 0 \qquad j \neq k \tag{A2}$$

It can be shown (ref. 6), that all orthogonal polynomials satisfy a three-term recurrence relation of the form

$$Q_{k+1} = (x - \alpha_k)Q_k - \beta_k Q_{k-1} \qquad k \geq 1 \tag{A3}$$

Thus by starting with $Q_1 = 1$ and $\beta_1 = 0$ an infinite set of orthogonal polynomials can be generated by means of equation (A3) if values for α_k and β_k are known. These can be determined from the orthogonality conditions (eqs. (A1) or (A2)). From the relation (A2) it follows that

$$\sum_{i=1}^{n} \tau_i^2 Q_k(x_i)Q_{k+1}(x_i) = 0 \tag{A4a}$$

and

$$\sum_{i=1}^{n} \tau_i^2 Q_{k+1}(x_i)Q_{k-1}(x_i) = 0 \tag{A4b}$$

When the recurrence relation (A3) is used to eliminate Q_{k+1}, there is obtained

$$\sum_{i=1}^{n} \tau_i^2 Q_k\Big[(x_i - \alpha_k)Q_k - \beta_k Q_{k-1}\Big] = 0 \tag{A5a}$$

$$\sum_{i=1}^{n} \tau_i^2\left[(x_i - \alpha_k)Q_k - \beta_k Q_{k-1}\right]Q_{k-1} = 0 \tag{A5b}$$

When the orthogonality condition (A2) is used, equations (A5a) and (A5b) reduce to

$$\sum_{i=1}^{n} \tau_i^2(x_i - \alpha_k)Q_k^2 = 0 \tag{A6a}$$

$$\sum_{i=1}^{n} \tau_i^2(x_i Q_k Q_{k-1} - \beta_k Q_{k-1}^2) = 0 \tag{A6b}$$

Solving equations (A6) for α_k and β_k gives

$$\alpha_k = \frac{\sum_{i=1}^{n} x_i \tau_i^2 Q_k^2}{\sum_{i=1}^{n} \tau_i^2 Q_k^2} \tag{A7a}$$

$$\beta_k = \frac{\sum_{i=1}^{n} x_i \tau_i^2 Q_k Q_{k-1}}{\sum_{i=1}^{n} \tau_i^2 Q_{k-1}^2} \tag{A7b}$$

Thus a set of orthogonal polynomials can be generated that are orthogonal over a finite set of discrete values of the variable x. Note that these values need not be equally spaced, a condition that is obviously necessary for stress-rupture data.

Scaling of Polynomial Argument

From the recurrence relation (A3) with $Q_1 = 1$, it follows that the leading term of $Q_{k+1}(x_i)$ is x_i^k. Therefore, depending on the values of x_i, the values of $Q_{k+1}(x_i)$ can become very large or very small. This procedure can lead to a loss of significant figures in performing the calculations. It is shown in reference 7, by comparison with the Chebyshov polynomials, that if x is scaled so that all the values of X_i lie between 2 and -2, the polynomial

values $Q_j(X_i)$ will all be of approximately uniform size. To perform this scaling, let x_{max} be the maximum value of $\log \sigma$ and x_{min} be the minimum value of $\log \sigma$; then let

$$X = A \log \sigma + B \tag{A8}$$

$$2 = Ax_{max} + B \tag{A9a}$$

$$-2 = Ax_{min} + B \tag{A9b}$$

and solving for A and B results in equations (9b).

It has been found in practice that scaling the values of x as indicated does indeed preserve the significance of the calculations.

Least-Squares Procedure

In terms of the orthogonal polynomials, equation (3) can be written

$$y = \sigma^q y_a + \tau \sum_{j=1}^{m+1} u_j Q_j(X) \tag{A10}$$

To find the best values of y_a and u_j that fit the data, the sum of the squares of the residuals is minimized. Thus let

$$S = \sum_{i=1}^{n} \left[y_i - \sigma_i^q y_a - \tau_i \sum_{j=1}^{n} u_j Q_j(X_i) \right]^2 \tag{A11}$$

Then in order to find the values of y_a and u_j that will make S a minimum, S is differentiated in turn with respect to y_a and each u_j, and the resulting equations are set equal to zero. When this is done, the following set of equations is obtained:

$$\left.\begin{array}{c}
a_0 y_a + a_1 u_1 + a_2 u_2 + \ldots + a_{m+1} u_{m+1} = c_0 \\
a_1 y_a + b_1 u_1 + 0 + \ldots + 0 = c_1 \\
a_2 y_a + 0 + b_2 u_2 + \ldots + 0 = c_2 \\
\vdots \quad \vdots \quad \vdots \\
a_{m+1} y_a + 0 + 0 + \ldots + b_{m+1} u_{m+1} = c_{m+1}
\end{array}\right\} \tag{A12}$$

where

$$
\left.
\begin{aligned}
a_0 &= \sum_{i=1}^{n} \sigma_i^{2q} \\
a_j &= \sum_{i=1}^{n} \sigma_i^{q}\tau_i Q_j(X_i) \qquad j = 1,2 \ldots m+1 \\
b_j &= \sum_{i=1}^{n} \tau_i^{2} Q_j^{2}(X_i) \qquad j = 1,2 \ldots m+1 \\
c_0 &= \sum_{i=1}^{n} \sigma_i^{q} y_i \\
c_j &= \sum_{i=1}^{n} \tau_i y_i Q_j(X_i) \qquad j = 1,2 \ldots m+1
\end{aligned}
\right\} \tag{A13}
$$

It is to be noted that the only nonzero elements in the coefficient matrix of equations (A12) are the diagonal elements and the elements of the first row and first column. All the other elements are zero because of the orthogonality properties of the polynomials used. This is one of the major advantages in using orthogonal polynomials. In the usual case of data fitting, all the elements of the first row and first column, except for the first element, would also be zero; and the equations would be completely uncoupled, each u_j being computed completely independent of the others, without the necessity of solving any sets of equations with the resultant loss of significant figures. In this particular case because of the added constant y_a, the equations are not completely uncoupled, but they are very nearly uncoupled and can readily be solved. Thus for any equation after the first

$$
u_j = \frac{c_j - a_j y_a}{b_j} \tag{A14}
$$

Substituting into the first equation and solving for y_a give immediately

$$
y_a = \frac{c_0 - \sum_{j=1}^{m+1} \frac{a_j c_j}{b_j}}{a_0 - \sum_{j=1}^{m+1} \frac{a_j^2}{b_j}} \tag{A15}
$$

APPENDIX B

FORTRAN IV PROGRAM

```
$ID    YAG1202 ERNEST ROBERTS, JR. - 140 M-S - PAX 6132
$LIBSIO         CONTINUE
$IBJOB          SOURCE
$IBFTC PRMTR1  LIST,REF,DECK
C          CREEP/STRESS-RUPTURE PARAMETER PROGRAM                       PRMT
C                                                                       PRMT
C          NOMENCLATURE IS AS FOLLOWS                                   PRMT
C                                                                       PRMT
C      DD          STANDARD DEVIATION                                   PRMT
C      KK          DEGREE OF FREEDOM                                    PRMT
C      KM          NUMBER OF VALUES OF M READ                           PRMT
C      KQ          NUMBER OF VALUES OF Q READ                           PRMT
C      KR          NUMBER OF VALUES OF R READ                           PRMT
C      KTA         NUMBER OF VALUES OF TTA READ                         PRMT
C      M           DEGREE POLYNOMIAL                                    PRMT
C      N           NUMBER OF DATA POINTS                                PRMT
C      PP          PARAMETER                                            PRMT
C      Q           STRESS EXPONENT                                      PRMT
C      QQ          POLYNOMIAL                                           PRMT
C      R           TEMPERATURE EXPONENT                                 PRMT
C      RATIO       ABS(Y-YY)/DD                                         PRMT
C      SIGMA       STRESS                                               PRMT
C      SIGQ        SIGMA**Q                                             PRMT
C      T           TIME                                                 PRMT
C      TA          TIME INTERCEPT                                       PRMT
C      TAU         SIGMA**Q*(TT-TTA)**R                                 PRMT
C      TAUSQR      TAU**2                                               PRMT
C      TIME        CALCULATED T (10.**YY)                               PRMT
C      TT          TEMPERATURE                                          PRMT
C      TTA         TEMPERATURE INTERCEPT                                PRMT
C      X           LOG SIGMA                                            PRMT
C      Y           LOG T                                                PRMT
C      YA          LOG TA                                               PRMT
C      YY          CALCULATED LOG T                                     PRMT
C                                                                       PRMT
C          ALL QUANTITIES IN COMMON WITH THIS PROGRAM AND THIS PAPER    PRMT
C          ARE REPRESENTED BY THE SAME SYMBOL, WITH REPEATED            PRMT
C      LETTERS INDICATING THE UPPER CASE AND GREEK LETTERS BEING SPELLED-PRMT
C      OUT.                                                             PRMT
C                                                                       PRMT
C          PROGRAM EXTRAPOLATES CREEP/STRESS-RUPTURE DATA USING A       PRMT
C      GENERALIZED PARAMETER                                            PRMT
C                PP=(Y/SIGMA**Q-YA)/(TT-TTA)**R,                        PRMT
C      SELECTS PARAMETER PRODUCING SMALLEST RESIDUAL AND OUTPUTS A      PRMT
C      COMPLETE TABLE. RESULTS OF ALL OTHER VALUES ARE SUMMARIZED IN    PRMT
C      A SHORTER TABLE.                                                 PRMT
C                                                                       PRMT
C          ************INPUT*************                               PRMT
C                                                                       PRMT
C      TITLE CARD, MODE CARD, AND FIVE (5) SETS OF DATA.  AT THE END OF PRMT
C      EACH SET OF DATA MUST BE A CARD WITH THE WORD 'END' IN THE FIRST PRMT
C      THREE COLUMNS.  ALL DATA CARDS (EXCEPTING TITLE AND MODE CARDS)  PRMT
C      MUST HAVE BLANKS IN THE FIRST THREE COLUMNS.  COLUMNS 73-80 ARE  PRMT
C      INGORED.                                                         PRMT
C                                                                       PRMT
C      TITLE - ANY ALPHAMERIC INFORMATION--HEADS EACH PAGE OF OUTPUT    PRMT
C                                                                       PRMT
C      MODE CARD - ONE OF THREE WORDS IN COLUMNS 1-6, 'LARSON', 'LINEAR',PRMT
C                  OR 'GENRAL'.  THIS CARD DEFINES 'KK', THE DEGREE OF  PRMT
C                  FREEDOM, USED IN CALCULATING GOODNESS OF FIT.        PRMT
C                                                                       PRMT
C      DATA SET 1--VALUES OF TTA TO BE INVESTIGATED--ONE PER CARD       PRMT
```

```
C                 FORMAT (3X,F10.0)--50 VALUES MAXIMUM
C
C     DATA SET 2--VALUES OF TEMPERATURE EXPONENT, R, TO BE INVESTIGATED
C                 ONE PER CARD--FORMAT (3X,F10.0)--20 VALUES MAXIMUM
C
C     DATA SET 3--VALUES OF STRESS EXPONENT,Q, TO BE INVESTIGATED
C                 ONE PER CARD--FORMAT (3X,F10.0)--20 VALUES MAXIMUM
C
C     DATA SET 4--DEGREES OF POLYNOMIAL, M, TO BE INVESTIGATED
C                 ONE PER CARD--FORMAT (3X,I2)--MAXIMUM VALUE NOT TO
C                 EXCEED 20--ZERO MAY NOT BE USED.
C
C     DATA SET 5--DATA POINTS IN THE ORDER TEMPERATURE, STRESS, AND
C                 TIME--ONE SET PER CARD--FORMAT (3X,3F10.0)
C                 THE VALUE OF STRESS IS AUTOMATICALLY DIVIDED BY 1000
C                 FOR ALL CALCULATIONS EXCEPT FINDING THE LOG STRESS.
C                 200 SETS MAXIMUM.
C
C            ************REPEAT************
C
C     EACH OF THE FIVE SETS OF DATA MUST BE FOLLOWED BY A CARD HAVING
C     THE WORD END IN THE FIRST THREE COLUMNS.
C     ALL DATA CARDS (EXCEPTING TITLE AND MODE CARDS) MUST HAVE THE
C     FIRST THREE COLUMNS BLANK.
C
C     WITHIN EACH SET, DATA MAY BE IN ANY ORDER.  IT WILL BE PROCESSED
C     IN THE ORDER PRESENTED TO THE MACHINE.
C
C     THE CALCULATIONS ARE PERFORMED IN FOUR (4) LOOPS.
C     GOING FROM INNERMOST TO OUTERMOST, THE QUANTITIES ARE VARIED
C     IN THE FOLLOWING ORDER
C            DEGREE POLYNOMIAL, M
C            VALUE OF TTA
C            TEMPERATURE EXPONENT, R
C            STRESS EXPONENT, Q
C
C     THE OUTPUT TABLES UTILIZE LESS THAN 120 COLUMNS ON THE PRINTER
C     AND EXPECT NO CARRIAGE CONTROLS OTHER THAN 1, 0, + AND BLANK.
C     A LINE COUNTER IS INCORPORATED TO LIMIT OUTPUT TO 60 LINES PER
C     PAGE. FOR EACH NEW PAGE THE TITLE AND APPROPRIATE COLUMN HEADINGS
C     ARE PRINTED. PROGRAM ENDS WITH A TRANSFER TO THE INITIAL READ.
C
C     PAGE COUNTING AND ERROR TRAPS MUST BE PROVIDED BY THE OPERATING
C     SYSTEM.
C
C     PROGRAM WITH IBSYS AND IOCSM WILL RUN ON A 16K MACHINE
C
C
      LOGICAL TRGGR1,TRGGR2,TRGGR3
C
      DIMENSION TITLE(12),TABLE(6,110),ITBLE(6,110)
C
      EQUIVALENCE (TABLE(1,1),ITBLE(1,1))
C
      COMMON /DATA/SIGMA(201),T(201),TT(201)
     1       /TRYS/M(21),Q(51),R(51),TTA(51)
     2       /FDATA/SIGQ(200),TAU(200),TAUSQR(200),X(200),XX(200),Y(200)
     3       /CALC/PP(200),RATIO(200),TIME(200),YY(200)
     4       /END/LND/N/N/DD/DD/DEGREE/DEGREE
     5       /PLYNML/OTHER1(4221),YA,OTHER2(63)
C
C
C            INPUT
C
    1 WRITE (6,9999)
      READ (5,9001) (TITLE(K),K=1,12)
```

```
      READ (5,9001) DEGREE
      K = 0
 10   K = K+1
      READ (5,9002) CHECK,TTA(K)
            IF (CHECK.NE.END) GO TO 10
      KTA = K-1
      K = 0
 15   K = K+1
      READ (5,9002) CHECK,R(K)
            IF (CHECK.NE.END) GO TO 15
      KR = K-1
      K = 0
 20   K = K+1
      READ (5,9002) CHECK,Q(K)
            IF (CHECK.NE.END) GO TO 20
      KQ = K-1
      K = 0
 25   K = K+1
      READ (5,9003) CHECK,M(K)
            IF (CHECK.NE.END) GO TO 25
      KM = K-1
      K = 0
 30   K = K+1
      READ (5,9004) CHECK,TT(K),SIGMA(K),T(K)
            IF (CHECK.NE.END) GO TO 30
      N=K-1
C
C           END OF INPUT
C
C           FIND LOG STRESS AND LOG TIME
C
            DO 100  K=1,N
      X(K)=ALOG10(SIGMA(K))+3.
      Y(K)=ALOG10(T(K))
 100        CONTINUE
C
C           INITIALIZE CONSTANTS
C
      DD1=1.E5
      LINES=51
      TRGGR3=.FALSE.
      NTRY=0
C
C           SCALE LOGS OF STRESS
C
      CALL SCALE
C
C           FIND HIGHEST DEGREE POLYNOMIAL
C
      MAX = 0
            DO 110  K=1,KM
      MAX = MAX0(MAX,M(K))
 110        CONTINUE
C
C           MAJOR LOOP - CALCULATES ALL Y(A)S AND RESIDUALS
C                        WRITES SUMMARY TABLE
C                        FINDS SMALLEST RESIDUAL
C
            DO 500  K5=1,KQ
C
C           CALCULATE SIGMA**Q
C
            DO 112 K=1,N
      SIGQ(K)=SIGMA(K)**Q(K5)
 112        CONTINUE
            DO 400  K4=1,KR
```

```
          DO 300  K3=1,KTA
C
C         CALCULATE TAU AND TAU**2
C
          DO 120  K=1,N
      TDIFF=ABS(TT(K)-TTA(K3))
          IF (TDIFF) 118,115,118
  115 TAU(K)=0.
          GO TO 119
  118 TAU(K)=SIGQ(K)*TDIFF**R(K4)
  119 TAUSQR(K) = TAU(K)**2
  120     CONTINUE
C
C         EVALUATE POLYNOMIALS
C
      CALL POLY(MAX)
C
          DO 200  K2=1,KM
C
C         DETERMINE Y(A)
C
      CALL YSUBA (M(K2))
C
C         CALCULATE THEORETICAL LOG TIMES AND TIMES
C
      CALL YTH(M(K2))
C
C         COMPUTE RESIDUAL
C
      CALL RESID(M(K2))
C
C         MAKE ONE ENTRY IN SUMMARY TABLE
C
      NTRY=NTRY+1
      TABLE(1,NTRY)=Q(K5)
      TABLE(2,NTRY)=R(K4)
      ITBLE(3,NTRY)=M(K2)
      TABLE(4,NTRY)=TTA(K3)
      TABLE(5,NTRY)=YA
      TABLE(6,NTRY)=DD
      TRGGR2=NTRY.EQ.2*LINES
          IF (TRGGR2) GO TO 170
          GO TO 190
C
C         OUTPUTS ONE PAGE OF SUMMARY TABLE
C
C
C         OUTPUT TITLE AND HEADINGS FOR SUMMARY TABLE
C
  170 WRITE (6,9005) (TITLE(K),K=1,12),DEGREE
          IF (LINES.EQ.51) WRITE (6,9006) KTA,KR,KQ,KM,N
      WRITE (6,9007)
      TRGGR1=NTRY.LE.LINES
      LIMIT=LINES
          IF (TRGGR1) LIMIT=NTRY
          DO 180 K=1,LIMIT
      WRITE (6,9008) (TABLE(I,K),I=1,2),ITBLE(3,K),(TABLE(I,K),I=4,6)
          IF (TRGGR1) GO TO 180
      KOL2=K+LINES
          IF (TRGGR2) GO TO 175
          IF (KOL2.GT.NTRY) GO TO 180
  175 WRITE (6,9009) (TABLE(I,KOL2),I=1,2),ITBLE(3,KOL2),
     1               (TABLE(I,KOL2),I=4,6)
  180     CONTINUE
      NTRY=0
      LINES=55
```

```
          IF (TRGGR3) GO TO 1000
C
C         SAVE VALUES PRODUCING SMALLEST RESIDUAL
C
 190      IF (DD1.LE.DD) GO TO 200
      M1 = M(K2)
      TTA1=TTA(K3)
      R1 = R(K4)
      Q1 = Q(K5)
      YA1 = YA
      DD1=DD
 200      CONTINUE
 300      CONTINUE
 400      CONTINUE
 500      CONTINUE
      TRGGR3=.TRUE.
          IF (NTRY.NE.0) GO TO 170
C
C         END MAJOR LOOP
C
C         OUTPUT OPTIMUM VALUES AND HEADING FOR FULL TABLE
C
 1000     CONTINUE
 1010 WRITE (6,9005) (TITLE(K),K=1,12),DEGREE
      LINES=3
 1020 WRITE (6,9010) Q1,R1,M1,TTA1,YA1,DD1
      LINES=LINES+5
 1030 WRITE (6,9011)
      LINES=LINES+3
C
C         CALCULATE THEORETICAL TIMES, RATIOS OF DIFFERENCES
C         TO RESIDUAL, AND VALUES OF THE PARAMETER, FOR THE
C         PARAMETER PRODUCING THE MINIMUM RESIDUAL
C
          DO 1035  K=1,N
      TDIFF=ABS(TT(K)-TTA1)
      SIGQ(K)=SIGMA(K)**Q1
          IF (TDIFF) 1032,1031,1032
 1031 TAU(K)=0.
          GO TO 1034
 1032 TAU(K)=SIGQ(K)*TDIFF**R1
 1034 TAUSQR(K) = TAU(K)**2
 1035     CONTINUE
      DD=DD1
      CALL POLY(M1)
      CALL YSUBA(M1)
      CALL YTH (M1)
      CALL RATIO1
      CALL PARAM
C
C         OUTPUT FULL TABLE
C
      K = 0
 1040 K = K+1
      WRITE (6,9012) TT(K),SIGMA(K),X(K),T(K),TIME(K),Y(K),YY(K),
     1               RATIO(K),PP(K)
      LINES=LINES+1
          IF (K.EQ.N) GO TO 1
          IF (LINES.LT.60) GO TO 1040
      WRITE (6,9005) (TITLE(KKK),KKK=1,12),DEGREE
      WRITE (6,9011)
      LINES=6
          GO TO 1040
C
C         END OF PROGRAM
C
```

```
C
C          FORMAT STATEMENTS FOR PROGRAM
C
C          FORMATS FOR INPUT
C
 9001 FORMAT (12A6)
 9002 FORMAT (A3,F10.0)
 9003 FORMAT (A3,I2)
 9004 FORMAT (A3,0PF10.0,3PF10.0,0PF10.0)
C
C          FORMATS FOR OUTPUT
C
C          TITLE (SKIPS TO NEW PAGE)
C
 9005 FORMAT(1H1,20X,12A6/1H ,30X,A6,10H PARAMETER/1H )
C
C          SUMMARY OF INPUT
C
 9006 FORMAT (1H ,10X,45HCREEP/RUPTURE PARAMETERS ARE INVESTIGATED FOR/
     11H ,I2,18H VALUE(S) OF T(A),,I3,25H TEMPERATURE EXPONENT(S),,I3,
     224H STRESS EXPONENT(S), AND,I3,14H POLYNOMIAL(S)/1H ,10X,5HUSING,
     3I4,12H DATA POINTS/1H )
C
C          HEADINGS FOR SUMMARY TABLE, ONE LINE OF SUMMARY TABLE
C
 9007 FORMAT (1H ,2(2X,1HQ,7X,1HR,6X,1HM,5X,4HT(A),5X,4HY(A),4X,
     1          8HSTD.DEV.,10X)/1H )
 9008 FORMAT (1H ,0PF5.2,F8.2,I5,F9.0,F10.2,1PE11.2)
 9009 FORMAT (1H+,58X,0PF5.2,F8.2,I5,F9.0,F10.2,1PE11.2)
C
C          OPTIMUM VALUES
C
 9010 FORMAT(1H 10X44HVALUES PRODUCING SMALLEST STANDARD DEVIATION/3H0Q=
     1F5.2,4H, R=F5.2,4H, M=I2,7H, T(A)=F6.0,7H, Y(A)=F9.3,11H, STD.DEV.
     2=1PE9.2/1H0)
C
C          HEADINGS FOR FULL TABLE, ONE LINE OF FULL TABLE
C
 9011 FORMAT (5H TEMP,4X,6HSTRESS,3X,3HLOG,6X,4HTIME,5X,6HCALCTD,5X,
     13HLOG,3X,8HCALC LOG,2X6HDEV/SD,3X,9HPARAMETER/1H ,8X,6H(*E-3),2X,
     26HSTRESS,14X,4HTIME,5X,4HTIME,4X,4HTIME/1H )
 9012 FORMAT (1H ,0PF5.0,F8.1,F8.3,2F10.1,3F9.3,1PE12.3)
C
 9999 FORMAT (1H1)
C
      END
```

```
$IBFTC PRMBLK  LIST,REF,DECK
C      SETS FIRST POLYNOMIAL TO UNITY AT ALL STATIONS AND STORES
C      ALPHAMERIC CODE WORDS
C
       BLOCK DATA
       COMMON /PLYNML/QQ(21,200),OTHERS(85)/END/END/NAMES/NAMES(2)
       DATA (QQ(1,K),K=1,200)/200*1./,END/3HEND/,
     1      (NAMES(K),K=1,2)/12HLARSONLINEAR/
       END


$IBFTC PARAM   LIST,REF,DECK
C      SUBROUTINE FOR EVALUATING THE PARAMETER AT EACH POINT
C
       SUBROUTINE PARAM
C
       COMMON /FDATA/SIGQ(200),TAU(200),OTHERS(600),Y(200)
     1        /CALC/PP(200),OTHER1(600)/N/N
     2        /PLYNML/OTHER2(4221),YA,OTHER3(63)
C
          DO 10  K=1,N
       PP(K) = (Y(K)-SIGQ(K)*YA)/TAU(K)
 10       CONTINUE
          RETURN
          END


$IBFTC YTH     LIST,REF,DECK
C      SUBROUTINE FOR CALCULATING TIMES AND LOG TIMES FROM THE PARAMETER
C
       SUBROUTINE YTH(M)
C
       COMMON /CALC/OTHERS(400),TIME(200),YY(200)
     1        /FDATA/SIGQ(200),TAU(200),OTHER1(800)
     2        /PLYNML/QQ(21,200),U(21),YA,OTHER2(63)
     3        /N/N
C
          DO 10  K=1,N
       YY(K) = 0.
 10       CONTINUE
       M1 = M+1
          DO 30 K=1,N
          DO 20  J=1,M1
       YY(K) = YY(K)+QQ(J,K)*U(J)
 20       CONTINUE
       YY(K) = TAU(K)*YY(K)+SIGQ(K)*YA
       TIME(K) = 10.**YY(K)
 30       CONTINUE
          RETURN
          END
```

```
$IBFTC RATIO1  LIST,REF,DECK
C      SUBROUTINE FOR CALCULATING RATIOS
C      OF INDIVIDUAL RESIDUALS TO ROOT-MEAN-SQUARE RESIDUAL
C
      SUBROUTINE RATIO1
C
      COMMON /FDATA/OTHERS(1000),Y(200)
     1        /CALC/OTHER1(200),RATIO(200),OTHER2(200),YY(200)
     2        /N/N/DD/DD
C
             DO 10  K=1,N
      RATIO(K) = ABS(Y(K)-YY(K))/DD
 10          CONTINUE
             RETURN
      END
```

```
$IBFTC RESID   LIST,REF,DECK
C      SUBROUTINE FOR CALCULATING RESIDUAL
C
C      THE RESIDUAL IS BASED ON THE LOG OF THE TIME.
C      IT IS DEFINED AS THE SQUARE ROOT OF THE SUM OF THE SQUARES OF
C      THE INDIVIDUAL RESIDUALS DIVIDED BY THE DIFFERENCE BETWEEN THE NUM
C      BER OF DATA POINTS AND THE DEGREES OF FREEDOM.  THE DEGREES OF
C      FREEDOM, KK, DEPENDS ON THE PARAMETER (SEE MAIN BODY OF REPORT).
C             KK=2 FOR LARSON-MILLER PARAMETER
C             KK=3 FOR LINEAR PARAMETER
C             KK=5 FOR GENERAL PARAMETER
C
C      DD = SQRT((Y-YY)**2/(N-M-KK))
C
      SUBROUTINE RESID(M)
C
      COMMON /FDATA/OTHERS(1000),Y(200)
     1        /CALC/OTHER1(600),YY(200)
     2        /DD/DD/N/N/DEGREE/DEGREE/NAMES/FAMES(2)
C
             IF (DEGREE.EQ.FAMES(2)) GO TO 20
             IF (DEGREE.EQ.FAMES(1)) GO TO 10
      KK = 5
             GO TO 30
 10   KK = 2
             GO TO 30
 20   KK = 3
 30   D = N-M-KK
      DD = 0.
             DO 40  K=1,N
      DD = DD+(Y(K)-YY(K))**2
 40          CONTINUE
      DD = SQRT(DD/D)
             RETURN
             END
```

```
$IBFTC YSUBA    LIST,REF,DECK
C       SUBROUTINE FOR EVALUATING Y(A)
C
C       THIS SUBROUTINE ALSO EVALUATES THE QUANTITIES, U, NECESSARY
C       FOR DETERMINING THE THEORETICAL LOG TIMES.
C
        SUBROUTINE YSUBA(M)
C
        COMMON /PLYNML/QQ(21,200),U(21),YA,A(21),B(21),C(21)
      1         /FDATA/SIGQ(200),TAU(200),TAUSQR(200),OTHERS(400),
      2                Y(200)/N/N
C
        A0 = 0.
        C0 = 0.
              DO 10  K=1,N
        A0 = A0+SIGQ(K)**2
        C0 = C0+SIGQ(K)*Y(K)
 10           CONTINUE
        M1 = M+1
              DO 20  J=1,M1
        A(J) = 0.
        B(J) = 0.
        C(J) = 0.
 20           CONTINUE
              DO 40  J=1,M1
              DO 30  K=1,N
        A(J) = A(J)+SIGQ(K)*TAU(K)*QQ(J,K)
        B(J) = B(J)+TAUSQR(K)*QQ(J,K)**2
        C(J) = C(J) + TAU(K)*Y(K)*QQ(J,K)
 30           CONTINUE
 40           CONTINUE
        SUM1 = 0.
        SUM2 = 0.
              DO 50  J=1,M1
        AOB = A(J)/B(J)
        SUM1 = SUM1+AOB*C(J)
        SUM2 = SUM2+AOB*A(J)
 50           CONTINUE
        YA =(C0-SUM1)/(A0-SUM2)
              DO 60  J=1,M1
        U(J) = (C(J)-A(J)*YA)/B(J)
 60           CONTINUE
              RETURN
              END
```

```
$IBFTC POLY     LIST,REF,DECK
C       SUBROUTINE FOR EVALUATING ORTHOGONAL POLYNOMIALS
C
C       ALL POLYNOMIALS UP TO MAXIMUM DESIRED DEGREE ARE EVALUATED
C       AT EACH DATA POINT
C
C       THE FIRST POLYNOMIAL IS IDENTICALLY EQUAL TO UNITY
C       THESE VALUES ARE STORED BY A BLOCK DATA SUBROUTINE
C
        SUBROUTINE POLY(M)
C
        COMMON /FDATA/OTHER1(400),TAUSQR(200),OTHER2(200),XX(200),
      1               OTHER3(200)
      2         /PLYNML/QQ(21,200),OTHERS(45),ALPHA(20),BETA(20)
      3         /N/N
C
        S1 = 0.
        S2 = 0.
              DO 10   K=1,N
        S1 = S1+XX(K)*TAUSQR(K)
        S2 = S2+TAUSQR(K)
 10           CONTINUE
        ALPHA(1) = S1/S2
              DO 20   K=1,N
        QQ(2,K) = XX(K)-ALPHA(1)
 20           CONTINUE
              IF (M.LE.1) RETURN
              DO 50   K=2,M
        S1 = 0.
        S2 = 0.
        S3 = 0.
        S4 = 0.
              DO 30   J=1,N
        D1 = TAUSQR(J)*QQ(K,J)
        D2 = D1*QQ(K,J)
        S1 = S1+XX(J)*D2
        S2 = S2+D2
        S3 = S3+XX(J)*D1*QQ(K-1,J)
        S4 = S4+TAUSQR(J)*QQ(K-1,J)**2
 30           CONTINUE
        ALPHA(K) = S1/S2
        BETA(K) = S3/S4
              DO 40   J=1,N
        QQ(K+1,J) = (XX(J)-ALPHA(K))*QQ(K,J)-BETA(K)*QQ(K-1,J)
 40           CONTINUE
 50           CONTINUE
              RETURN
              END
```

```
$IBFTC SCALE    LIST,REF,DECK
C       SUBROUTINE FOR SCALING LOGS OF STRESS
C
C       THE SCALED VALUES LIE IN THE REGION -2 TO 2
C
        SUBROUTINE SCALE
C
        COMMON /FDATA/OTHER1(600),X(200),XX(200),OTHER2(200)/N/N
C
        BIG = 0.
        SMALL = 1.E5
              DO 10  K=1,N
        BIG = AMAX1(BIG,X(K))
        SMALL = AMIN1(SMALL,X(K))
 10           CONTINUE
        A = 4./(BIG-SMALL)
        B=2.*(BIG+SMALL)/(BIG-SMALL)
              DO 20  K=1,N
        XX(K) = A*X(K)-B
 20           CONTINUE
              RETURN
              END
```

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TABLE I. - CALCULATION OF POLYNOMIALS FOR THEORETICAL DATA FOR THIRD DEGREE POLYNOMIAL

[Temperature intercept, T_a, 600° F.]

1	2	3	4	5	6	7	8	9	10	11	12
Index, i	Temperature, T, °F	Time, t, hr	Stress, σ, psi	log t	log σ	$\sigma^q(T-Ta)^r$	Scaled log σ, X	Polynomial			
								Q_1	Q_2	Q_3	Q_4
1	1100	4954.68	56 300	3.69501	4.75051	500	2.0	1	1.3619	-0.19594	-0.50845
2	1100	11365.9	19 800	4.05560	4.29666	500	1.3806	1	.30341	-1.6875	-.57205
3	1200	625.342	30 300	2.79612	4.48144	600	1.6328	1	.85576	-1.4583	1.7195
4	1200	2908.	5 080	3.46359	3.70586	600	.57433	1	-.80869	1.5741	-1.2980
5	1300	117.371	12 900	2.06956	4.11059	700	1.1267	1	.18925	2.5067	-1.0745
6	1300	1340.	778	3.12710	2.89098	700	-.53777	1	-2.2709	-.90880	.92564
7	1400	34.4856	4 190	1.53764	3.62221	800	.46017	1	2.8030	.015445	-.77354
8	1400	995.25	66	2.99793	1.81954	800	-2.0	1	.51879	-.78251	-.98133

TABLE II. - INTERMEDIATE CALCULATIONS FOR THEORETICAL DATA FOR THIRD DEGREE POLYNOMIAL

[Temperature intercept, T_a, 600° F.]

1	2	3	4	5	6	7
Index, j	α	β	a	b	c	u
0	-------	-------	8.0	--------	23.743	------------
1	0.27092	0.	5200.	3.48×10^6	14897.	-9.9146×10^{-3}
2	-.57813	1.6548	786.18	5.7589	2616.	-8.4266×10^{-4}
3	.28432	1.3315	465.68	7.6678	3796.9	-8.1771×10^{-5}
4	.41260	.80618	286.01	6.1816	2694.6	-3.6513×10^{-6}

TABLE III. - FIT FOR SEVERAL VALUES OF LINEAR PARAMETER FOR THEORETICAL DATA

Degree of polynomial	Temperature, T_a	Variable, y_a	Deviation
2	500	10.54	0.008049
3	500	10.54	.009786
4	500	10.55	.010660
2	600	9.49	.004859
3	600	9.50	.000002
4	600	9.50	.000003
2	700	8.44	.015412
3	700	8.46	.013937
4	700	8.45	.014469

TABLE IV. - COMPOSITION OF STEELS RECEIVED FROM GERMAN COOPERATIVE LONG-TIME CREEP PROGRAM

[As-received, 20-mm-diam. bar stock.]

Element	Composition, percent		
	Steel		
	C (23b CK)	P (14a PA)	K (27b KK)
Carbon	0.065	0.270	0.068
Silicon	.47	.26	.45
Manganese	.60	.60	.73
Chromium	17.24	2.62	16.14
Molybdenum	2.08	.27	2.10
Columbium and tantalum	.02	Trace	.44
Nickel	11.90	.14	13.12
Titanium	.39	Trace	Trace
Vanadium	.10	.26	.05
Tungsten	Less than 0.005	Trace	Trace

TABLE V. - NASA RUPTURE DATA

(a) Steel K (27b KK)

Temperature, T, °F	Stress, σ, psi	Time, t, hr	Temperature, T, °F	Stress, σ, psi	Time, t, hr	Temperature, T, °F	Stress, σ, psi	Time, t, hr
1022.00	77 000.000	1.500	1600.00	20 000.000	0.400	[a]1112.00	60 000.000	12.900
[a]1022.00	72 500.000	13.800	1560.00	20 000.000	1.900	[a]1110.00	60 000.000	34.
[a]1022.00	72 000.000	10.	1520.00	20 000.000	4.450	[a]1080.00	60 000.000	52.200
[a]1022.00	70 000.000	36.700	[a]1480.00	20 000.000	23.700	[a]1080.00	60 000.000	37.400
[a]1022.00	68 000.000	60.400	[a]1460.00	20 000.000	25.500	[a]1050.00	60 000.000	239.
[a]1022.00	66 000.000	73.300	[a]1440.00	20 000.000	38.	[a]1030.00	60 000.000	445.
[a]1022.00	66 000.000	107.600	[a]1400.00	20 000.000	136.800	[a]1022.00	60 000.000	989.900
[a]1022.00	65 000.000	201.300	[a]1360.00	20 000.000	394.800	[a]1020.00	60 000.000	817.500
[a]1022.00	62 500.000	250.400	[a]1340.00	20 000.000	704.600	1040.00	75 000.000	.330
[a]1022.00	60 000.000	990.	[a]1320.00	20 000.000	1212.	1022.00	75 000.000	5.850
[a]1022.00	60 000.000	817.500	1320.00	40 000.000	2.700	[a]1000.00	75 000.000	15.600
[a]1022.00	55 000.000	3 680.	1290.00	40 000.000	7.500	[a]980.00	75 000.000	46.500
1112.00	68 000.000	.750	[a]1260.00	40 000.000	15.200	[a]960.00	75 000.000	138.
1112.00	65 000.000	2.250	[a]1230.00	40 000.000	44.400	[a]940.00	75 000.000	542.
1112.00	62 500.000	4.300	[a]1170.00	40 000.000	377.	[a]920.00	75 000.000	579.600
[a]1112.00	60 000.000	13.900	[a]1140.00	40 000.000	1417.	[a]1120.00	50 000.000	186.100
[a]1112.00	57 500.000	22.700	[a]1125.00	40 000.000	2110.	[a]1200.00	40 000.000	130.200
[a]1112.00	55 000.000	51.500	[a]1112.00	40 000.000	5367.	[a]1280.00	30 000.000	132.700
[a]1112.00	52 500.000	147.500	1200.00	60 000.000	.610	[a]1340.00	25 000.000	125.800
[a]1112.00	50 000.000	283.	1170.00	60 000.000	1.250	[a]1500.00	15 000.000	51.300
[a]1112.00	45 000.000	1 020.	1150.00	60 000.000	4.400	[a]1560.00	12 000.000	41.700
[a]1112.00	43 000.000	1 579.	1140.00	60 000.000	4.500	[a]1580.00	10 000.000	32.400
1112.00	37 000.000	13 140.	[a]1120.00	60 000.000	10.900	[a]1540.00	10 000.000	148.200

(b) Steel C (23b CK)

Temperature, T, °F	Stress, σ, psi	Time, t, hr	Temperature, T, °F	Stress, σ, psi	Time, t, hr	Temperature, T, °F	Stress, σ, psi	Time, t, hr
[a]1600.00	5 000.000	570.200	[a]1230.00	30 000.000	175.700	[a]1202.00	36 000.000	68.600
[a]1620.00	5 000.000	186.600	[a]1250.00	30 000.000	103.500	[a]1202.00	38 000.000	59.300
[a]1660.00	5 000.000	156.800	[a]1280.00	30 000.000	58.100	[a]1202.00	42 000.000	24.400
[a]1680.00	5 000.000	91.600	1292.00	30 000.000	21 300.	[a]1202.00	44 000.000	14.500
[a]1700.00	5 000.000	62.700	[a]1310.00	30 000.000	22.500	[a]1202.00	45 000.000	22.900
[a]1740.00	5 000.000	40.500	[a]1112.00	40 000.000	667.900	1202.00	46 000.000	7.
[a]1780.00	5 000.000	10.600	[a]1120.00	40 000.000	785.400	1202.00	48 000.000	2.850
[a]1425.00	10 000.000	1690.	[a]1150.00	40 000.000	266.700	1202.00	49 000.000	2.550
[a]1450.00	10 000.000	550.300	[a]1170.00	40 000.000	127.800	1202.00	50 000.000	1.470
[a]1480.00	10 000.000	270.	[a]1202.00	40 000.000	44.100	[a]1292.00	18 000.000	859.700
[a]1500.00	10 000.000	170.	[a]1202.00	40 000.000	74.	[a]1292.00	23 000.000	194.600
[a]1520.00	10 000.000	128.500	[a]1210.00	40 000.000	40.500	[a]1292.00	25 000.000	75.
[a]1560.00	10 000.000	40.	[a]1220.00	40 000.000	37.800	[a]1292.00	28 000.000	34.600
[a]1570.00	10 000.000	31.500	[a]1240.00	40 000.000	17.200	[a]1292.00	29 000.000	31.
[a]1600.00	10 000.000	15.800	1270.00	40 000.000	4.500	[a]1292.00	32 000.000	13.300
1650.00	10 000.000	5.250	1280.00	40 000.000	1.200	[a]1292.00	33 000.000	19.800
1700.00	10 000.000	1.750	1292.00	40 000.000	1.300	[a]1292.00	34 000.000	10.400
[a]1202.00	20 000.000	3307.	1300.00	40 000.000	.800	1292.00	36 000.000	2.750
[a]1260.00	20 000.000	667.400	[a]1112.00	34 000.000	2 274.	1292.00	37 000.000	7.600
[a]1290.00	20 000.000	255.	[a]1112.00	43 000.000	363.100	1292.00	38 000.000	1.650
[a]1292.00	20 000.000	347.100	[a]1112.00	46 000.000	233.900	[a]1060.00	60 000.000	42.500
[a]1292.00	20 000.000	363.	[a]1112.00	46 000.000	261.400	[a]1300.00	25 000.000	89.600
[a]1320.00	20 000.000	180.400	[a]1112.00	48 000.000	183.100	[a]1360.00	19 000.000	95.
[a]1360.00	20 000.000	82.	[a]1112.00	50 000.000	84.500	[a]1430.00	15 000.000	71.400
[a]1400.00	20 000.000	28.900	[a]1112.00	52 000.000	65.600	[a]1480.00	12 000.000	147.900
1440.00	20 000.000	9.	[a]1112.00	54 000.000	39.300	[a]1570.00	8 000.000	104.
1480.00	20 000.000	2.500	[a]1112.00	57 000.000	23.300	[a]1630.00	6 000.000	140.900
[a]1112.00	30 000.000	4258.	[a]1202.00	25 000.000	1 074.	[a]1140.00	34 000.000	1077.
[a]1160.00	30 000.000	1110.	[a]1202.00	34 000.000	199.400	[a]1320.00	15 000.000	1505.
[a]1180.00	30 000.000	696.300	[a]1202.00	35 000.000	124.300	[a]1480.00	8 000.000	2237.
[a]1202.00	30 000.000	350.				[a]1540.00	6 000.000	1258.

[a]Data point used in parametric analysis.

TABLE V. - Concluded. NASA RUPTURE DATA

(c) Steel P (14a PA)

Temperature, T, °F	Stress, σ, psi	Time, t, hr	Temperature, T, °F	Stress, σ, psi	Time, t, hr	Temperature, T, °F	Stress, σ, psi	Time, t, hr
932.00	65 000.000	3.800	[a]1250.00	10 000.000	19.200	[a]740.00	90 000.000	57.100
[a]932.00	60.000.000	14.150	[a]1220.00	10 000.000	42.	[a]785.00	80 000.000	84.
[a]932.00	60 000.000	14.400	[a]1180.00	10 000.000	167.	[a]820.00	70 000.000	195.800
[a]932.00	57 500.000	10.	[a]1170.00	10 000.000	203.400	[a]880.00	60 000.000	120.
[a]932.00	55 000.000	18.900	[a]1140.00	10 000.000	608.	[a]932.00	50 000.000	103.500
[a]932.00	52 500.000	51.	[a]1090.00	10 000.000	2639.	[a]1022.00	30 000.000	186.700
[a]932.00	40 000.000	623.	1100.00	40 000.000	1.300	[a]1050.00	25 000.000	123.500
[a]932.00	30 000.000	7 592.	1080.00	40 000.000	2.200	[a]1090.00	20 000.000	79.500
932.00	27 000.000	11 410.	1060.00	40 000.000	4.300	[a]1090.00	20 000.000	112.400
1022.00	58 000.000	.580	1050.00	40 000.000	6.800	[a]1120.00	16 000.000	183.500
1022.00	55 000.000	.717	1040.00	40 000.000	7.400	[a]1160.00	13 000.000	100.300
1022.00	50 000.000	1.280	[a]1020.00	40 000.000	22.500	[a]1230.00	8 000.000	97.900
1022.00	47 000.000	2.450	[a]1010.00	40 000.000	20.100	[a]1290.00	5 000.000	139.700
1022.00	47 000.000	6.200	[a]1000.00	40 000.000	63.300	[a]740.00	80 000.000	996.600
1022.00	45 000.000	3.500	[a]990.00	40 000.000	51.200	[a]780.00	70 000.000	1122.
1022.00	42 500.000	6.300	[a]980.00	40 000.000	80.600	[a]830.00	60 000.000	948.800
[a]1022.00	40 000.000	22.500	[a]960.00	40 000.000	192.100	[a]880.00	50.000.000	599.
[a]1022.00	37 500.000	12.	[a]940.00	40 000.000	427.900	[a]932.00	35 000.000	1902.
[a]1022.00	25 000.000	382.200	[a]930.00	40 000.000	623.	[a]980.00	30 000.000	754.800
1415.00	10 000.000	.170	[a]900.00	40 000.000	2572.	[a]1000.00	25 000.000	970.700
1340.00	10 000.000	1.500	932.00	70 000.000	1.400	[a]1030.00	20 000.000	1084.
1315.00	10 000.000	3.700	897.00	70 000.000	5.800	[a]1070.00	16 000.000	804.800
1290.00	10 000.000	6.100	[a]860.00	70 000.000	31.200	[a]1150.00	8 000.000	948.500
						[a]1220.00	5 000.000	960.

[a]Data point used in parametric analysis.

TABLE VI. - GERMAN RUPTURE DATA

Temperature, T, °F	Stress, σ, psi	Time, t, hr
Steel K (27b KK)		
1022.00	76 899.999	0.100
1022.00	66 899.999	160.
1022.00	55 500.000	2 000.
1112.00	72 500.000	.100
1112.00	64 000.000	10.
1112.00	55 500.000	35.
1112.00	44 100.000	2 100.
1112.00	28 400.000	52 000.
Steel C (23b CK)		
1112.00	14 200.000	60 000.
1112.00	17 800.000	30 000.
1112.00	28 400.000	3 500.
1112.00	28 400.000	3 000.
1112.00	28 400.000	2 200.
1112.00	35 600.000	1 200.
1112.00	44 100.000	520.
1112.00	51 200.000	150.
1112.00	59 800.000	.100
1202.00	11 400.000	82 790.
1202.00	14 200.000	15 000.
1202.00	17 800.000	6 500.
1202.00	22 800.000	1 800.
1202.00	28 400.000	550.
1202.00	35 600.000	124.
1202.00	42 700.000	5.
1202.00	52 600.000	.100
1292.00	11 400.000	30 000.
1292.00	11 400.000	20 000.
1292.00	13 900.000	4 500.
Steel C (23b CK)		
1292.00	17 800.000	1 100.
1292.00	21 400.000	300.
1292.00	21 400.000	250.
1292.00	27 000.000	180.
1292.00	27 000.000	140.
1292.00	47 000.000	.100
Steel P (14a PA)		
932.00	84 000.000	0.100
932.00	75 500.000	.100
932.00	78 399.999	2.
932.00	55 500.000	150.
932.00	44 100.000	1 700.
932.00	34 200.000	2 600.
932.00	27 000.000	16 000.
932.00	22 800.000	22 000.
932.00	17 100.000	100 000.
1022.00	72 599.999	.100
1022.00	69 699.999	.100
1022.00	65 500.000	1.200
1022.00	59 800.000	1.500
1022.00	35 600.000	150.
1022.00	27 000.000	300.
1022.00	22 800.000	400.
1022.00	22 800.000	900.
1022.00	17 100.000	2 100.
1022.00	13 900.000	6 500.
1022.00	13 900.000	8 000.
1022.00	11 100.000	10 000.
1022.00	8 830.000	68 000.

TABLE VII. - CREEP DATA FOR COLUMBIUM ALLOY FS-85

Temperature, T, °F	Stress, σ, psi	Time, t, hr		
		1-Percent creep	2-Percent creep	5-Percent creep
2005	25 000	0.6	3.0	6.1
1900	25 000	26.	33.	45.
1790	25 000	210.	257.	332.
2175	18 000	4.9	7.8	13.
2400	10 000	3.4	5.7	10.8
2300	10 000	25.4	41.	68.
2200	10 000	54.	84.	133.
2100	10 000	355.	500.	765.
2100	10 000	380	570.	875.
2000	10 000	775.	1325.	2175.
2000	10 000	900.	1420.	------
2000	8 500	2480.	------	------
2575	6 000	5.6	10.	22.2
2200	6 000	425.	710.	1370.
2800	4 000	3.4	6.4	13.5
2620	4 000	14.4	26.	56.
2200	4 000	1140.	------	------
2900	3 000	2.6	5.4	13.8
3000	2 000	4.6	9.5	33.2
2450	2 000	------	------	950.

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